

Portraits and Rays

$$E: \begin{matrix} S' & \rightarrow & S' \\ x & \mapsto & z_x \end{matrix} \quad \curvearrowright \quad \mathcal{F}$$

• $f_c(z) = z^2 + c$, $c \in \mathcal{M}$, $\gamma: \mathbb{B}/\mathbb{Z} \rightarrow \mathcal{J}(f_c)$
 $\theta \mapsto \text{landing pt of } R_\theta$

$\{z_0, z_1, \dots, z_{n-1}\}$ superparach orbit in $\mathcal{J}$,

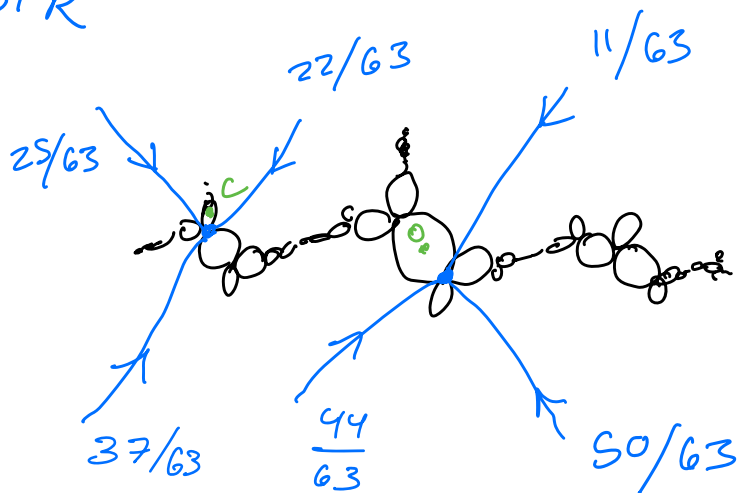
$\{\gamma^{-1}(z_0), \dots, \gamma^{-1}(z_{n-1})\}$, $\gamma^{-1}(z_i) \subset S^1$

Ex: Basilica Tuned by
Fat Rabbit - BTbFR

$$c = \frac{e^{2\pi i/3}}{9} - 1 \quad \text{parabolic orbit}$$

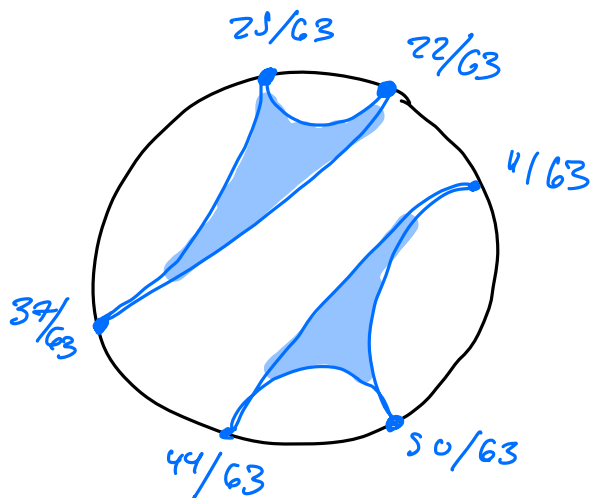
$$\frac{11}{63} \rightarrow \frac{22}{63} \rightarrow \frac{44}{63} \rightarrow \frac{25}{63}$$

$$\frac{80}{63} \rightarrow \frac{37}{63}$$



$$\left\{ \left\{ \frac{11}{63}, \frac{44}{63}, \frac{50}{63} \right\}, \right.$$

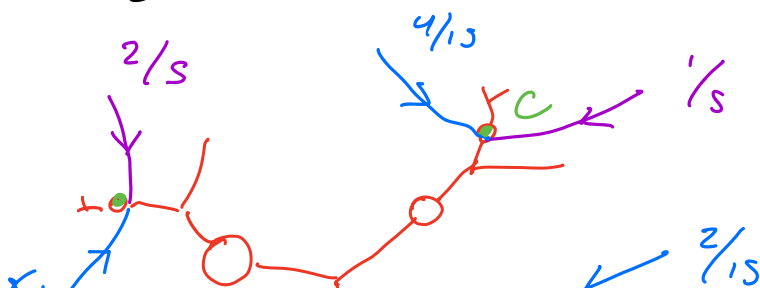
$$\left. \left\{ \frac{22}{63}, \frac{25}{63}, \frac{37}{63} \right\} \right\}$$



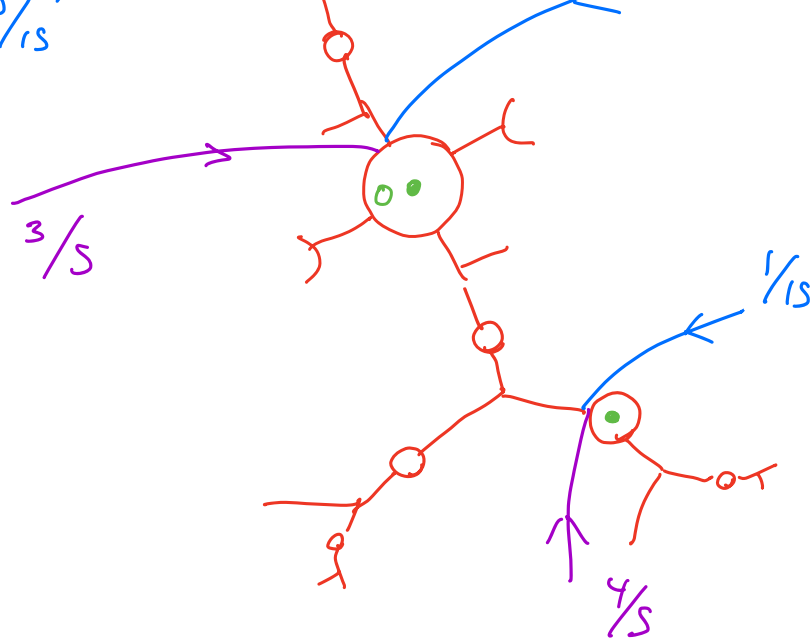
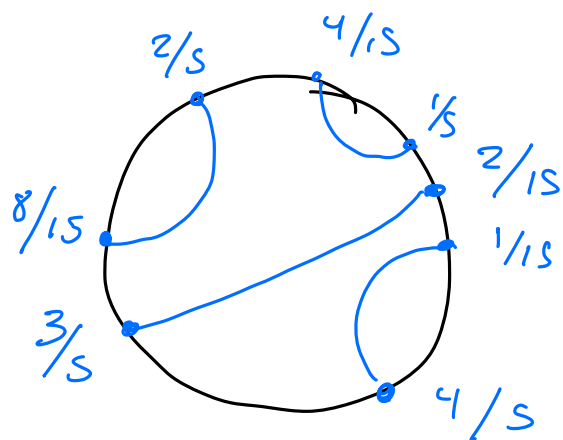
Ex: Kokopelli

$$c = -0.15652 + 1.03225i$$

$$\frac{1}{15} \rightarrow \frac{2}{15} \rightarrow \frac{4}{15} \rightarrow \frac{8}{15}$$



$$\frac{1}{5} \rightarrow \frac{2}{5} \rightarrow \frac{4}{5} \rightarrow \frac{3}{5}$$



$$\left\{ \left\{ \frac{1}{5}, \frac{4}{5} \right\}, \left\{ \frac{2}{5}, \frac{3}{5} \right\}, \left\{ \frac{1}{5}, \frac{4}{5} \right\}, \left\{ \frac{2}{5}, \frac{3}{5} \right\} \right\}$$

Sectors : region bounded by 2 rays meeting at the same point

Def. Orbit portrait : $A = \{A_1, \dots, A_k\}$, $A_{k+1} = A_1$,

- 1) A_i are pairwise disjoint finite subset of S^1
- 2) $E: A_i \xrightarrow{\sim} A_{i+1}$ preserving cyclic order
- 3) A_i are pairwise unlinked: if $i \neq j$, then A_i is contained in one component of
 $S^1 \setminus A_j$ complementary arc of A_j

Leave 1) Every comp. arc of A_i is mapped

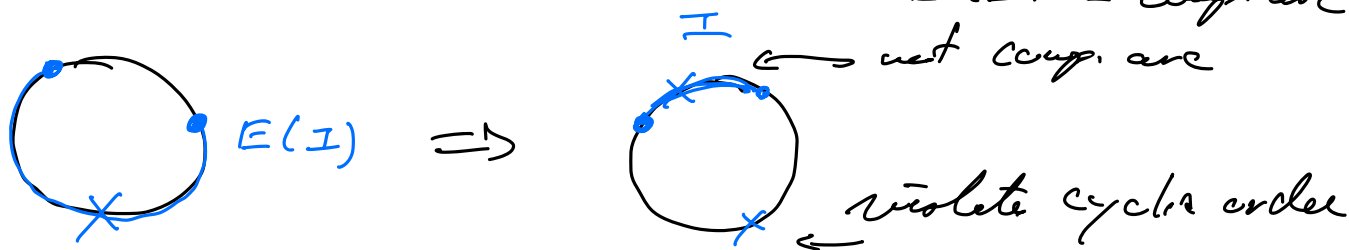
bijectively maps E to a complementary arc of A_{i+1} except for one, called the critical arc.

2) Maps E , the crit. arc of A_i covers all of the complementary arcs of A_{i+1} except for one arc, called the critical value arc of A_{i+1} , which is covered twice.

Ex: A run for oriented cycle σ of f :
critical arc is the one whose sector contains 0
crit. value arc: sector contains c

Proof. 1) No complementary arc has length $\frac{1}{2}$

• I is comp. arc of A_i , $l(I) < \frac{1}{2} \Rightarrow E(I) \cap A_{i+1}$ contains only the midpoint
 $E(I) \Rightarrow$ comp. arc
 not comp. arc



• If all $l(I) < \frac{1}{2} \Rightarrow E(I)$ don't overlap,

but $\sum l(I) = 1$ $\sum l(E(I)) = 2$. X

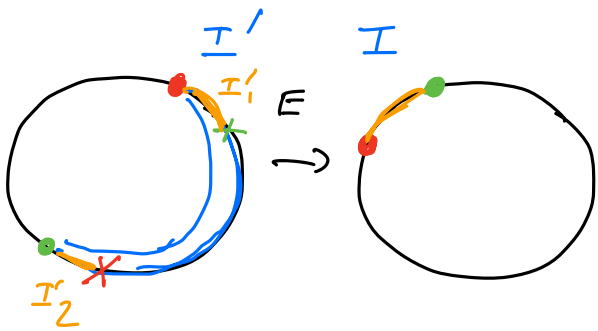
- $\Rightarrow \exists!$ crit. arc of A_i , $l(I) > \frac{1}{2}$.
- 2) I crit. arc, $S' = I \cup \bigcup I_j$ ← non-crit. arcs

• $E(I) = S'$, I_j are inj, so the $E(I_j)$ are covered
 ✓ covered by I_j , are by I . Since E is 2:1,
 $S' \setminus \bigcup_j E(I_j)$ is covered twice by I .

because $\exists!$ shortest complementary arc among all
 A_i called the characteristic arc. It is the
 intersection of all crit. value arcs.

Ex: see 4's. For a supercritical,
 char. arc is bounded by the char angles.
Proof. • I arc of min length l . $\Rightarrow I$ is
 a crit. value arc, because otherwise, $I = E(I')$,
 $l(I') = l/2$.

- I crit. arc $I' \xrightarrow{2:1} I$, $I \rightarrow S'$



• I is min, so
 part of $\bigcup A_i$ is I ,
 no part of $\bigcup A_i$ is $E^{-1}(I)$.

- $E^{-1}(I) = I_1' \sqcup I_2' \subset I'$ are the preimages
- Munking $\Rightarrow A_j \subset \underbrace{S' \setminus I'}_{\text{length } \frac{1-l(I)}{2}} \text{ or } A_j \subset \underbrace{I'}_{\substack{\subset \\ \text{length } \frac{1-l(I)}{2}}} \setminus E^{-1}(I)$
- All non-cent. arcs of A_j are in one of these two cases of length $\frac{1-l}{2}$.
- Lemma Ξ : all non-cent. sub-arcs of A_{j+1} are in $S' \setminus I$, have length $\leq 1-l$, so the cent. sub-arc of $A_j \supset I$. Munking. ✓

Lemma. A portrait, $|A_i| > 1$. Then either

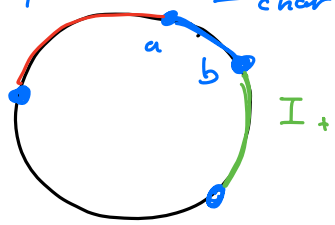
(Primitive) $E^u|_{A_i} = \text{id}|_{A_i}$ and $|A_i| = 2$ or

(Satellite) $\bigcup_{i=1}^u A_i$ form a single orbit under E

Ex: Primitive: Rokhlin Satellite: BTBFR

Proof. WTS: # orbits of $\bigcup_{i=1}^u A_i$ under E is ≤ 2

A_i I_- I_{char} I_+ $\leftarrow$ char. arc, cent. sub-arc of A_i ; $a, b \in A_i$

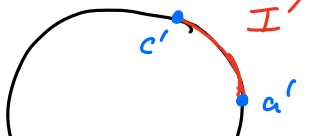


• Suppose # cycles ≥ 3 .

$\Rightarrow |A_i| \geq 3$, $I_+ \neq I_-$ adjacent comp.

arcs, $l(I_+) \geq l(I_-)$.

A_{i-u}



I_- not cent. sub-arc, so $I_- = E^u(I')$

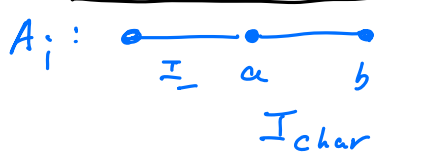
for a comb. relation $\mathbb{I}'$, $E^u(a') = \alpha$

$$l(\mathbb{I}_-) = 2^u l(\mathbb{I}'),$$

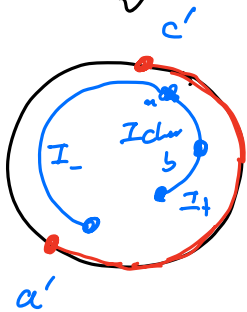
cycles $\geq 3 \Rightarrow a, b$ are not in the same cycle $\Rightarrow E^l(a') \neq b \text{ H.l.}$, so $\mathbb{I}' \neq \mathbb{I}_{\text{char.}}$

$\Rightarrow \mathbb{I}_{\text{char.}} \not\subseteq \mathbb{I}'$ on both sides since $\mathbb{I}'$ is

A_i - u_i : $\overline{c' \xrightarrow{\mathbb{I}'} a'}$ comb. relation.



$\Downarrow$



Since $l(\mathbb{I}_-) > l(\mathbb{I}')$,

$\mathbb{I}_- \not\subseteq \mathbb{I}'$ *cutting + overlaps*
 $\Rightarrow a', c' \in \mathbb{I}_-$

$\Rightarrow \mathbb{I}_+ \subset \mathbb{I}'$, but

$$l(\mathbb{I}_+) \leq l(\mathbb{I}') < l(\mathbb{I}_-). \quad \times$$